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The Model for Predicting the Radius of Curvature of the Neck Forming on Cylindrical Specimens during Tensile Testing

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Abstract. The article is devoted to the testing of cylindrical specimens for tension. The main tendency of tensile tests is the improvement of methods for determining equivalent stresses acting in specimens while deformation localizes in the neck, i.e. methods for determining the hardening curves of materials up to the fracture. The most reliable way to study the rheological properties of materials is the direct measurement of the neck profile, namely the determination of the specimen diameter in the minimum cross-section of the neck and the radius of its curvature. However, direct measurements of the neck are associated with certain difficulties. They include a small number of measurements corresponding to different instants of the test, as well as inaccurate determination of the radius of the neck curvature due to the subjective approach to the measurement process. The measurements become more accurate if you use some equation for the analytical description of the neck profile and perform the measurements using optical methods during the test. However, such processing algorithms are often inaccessible to researchers. In this regard, this work proposes a model for predicting the radius of the neck curvature that is formed during tensile testing the cylindrical specimens. This model relies on the volume conservation law and the analytical description of the neck profile using the one-parameter equation. To calculate the radius of the neck curvature at each instant of the test it is necessary to know only the current values of elongation and the minimum specimen diameter. The effectiveness of the proposed mathematical model is evaluated using the computer simulation. The simulation results show that the use of predicted radius of the neck curvature allows calculating the equivalent stresses close to proper ones without the need of the direct neck profile measurement.

1. Introduction

One of the most common methods for studying the mechanical properties of materials is the tension of cylindrical specimens [1, 2]. During the test, it is possible to determine the mechanical characteristics of materials such as yield strength, tensile strength, percentage extension and percentage reduction of area, as well as a number of other characteristics [3–7]. However, during standard tensile tests it is not possible to achieve large degrees of uniform deformation because the softening of the specimen caused by a decrease in its cross-sectional area begins to prevail over the strain hardening of the material. As a result, the deformation gradually localizing in a narrow area called a neck. Due to the complexity of processing the experimental data corresponding to the stage of concentrated deformation of the specimen, tensile tests have not been widely used to study the rheological properties of materials. However, tensile tests are very promising for their wider use.



To define hardening curves of the materials based on the results of the tensile tests, it is necessary to calculate the equivalent stresses taking into account the inhomogeneous distribution of the axial tensile stresses in the neck area [8–11]:

$$\sigma_{eq} = \frac{P}{F \cdot K}, \quad (1)$$

where P is the tensile force, F is the cross-sectional area of the specimen and K is the correction coefficient that takes into account the inhomogeneous stress state of the material. The values of equivalent stresses have to be associated with the values of equivalent strains determined by the formula:

$$\varepsilon_{eq} = 2 \ln \frac{d_0}{d}, \quad (2)$$

where d_0 is the original diameter of the specimen gauge length and d is the current value of the diameter corresponding to minimum cross-section of the specimen.

The main difficulty in defining the hardening curves of the material is the determination of the correction coefficient K that may correspond to the Bridgman [8], Davidenkov-Spiridonova [9] or Ostsemin [10, 11] models. The Ostsemin model is the most advanced among them. According to it, the formula to calculate the correction coefficient is as follows:

$$K = \left(1 + 2 \frac{R}{d}\right) \cdot \ln \left(1 + \frac{d}{2R}\right), \quad (3)$$

where R is the radius of the neck curvature in the minimum section of the specimen.

The measurements of the radius R are associated with certain difficulties. Usually to measure the neck profile the specimen is removed from the grips of a test setup. Then the radius of the neck curvature is determined using an instrumental microscope [12], digital image correlation method [13] or using a set of templates and enlarged specimen images obtained with a projector [8]. In this case, the number of neck measurements is very limited and the test procedure is long-time and hard. Moreover, the neck profile measurements are subjective because a researcher visually selects the circle to approximate the neck profile.

The use of some equation to describe the neck profile is the most advanced measuring procedure. Knowing such an equation, one can calculate the desired radius of the neck curvature based on methods of mathematical analysis [14, 15]. At the same time, the specimen can stay in the grips of the test setup during the neck profile measurement that is performed with the use of the photography [16] or the video recording of the testing process [17, 18]. However, the algorithms for the neck images processing are closed and inaccessible to a wide circle of researchers.

A radically different approach is proposed in [15, 19], where the authors claim that the radius of the neck curvature is not depended on the properties of the material. They propose the statistical models for calculating the equivalent stresses according to a large number of experiments. However, the results of another study [20] show that the material properties affect the radius of the neck curvature and the assumption proposed in [15, 19] is incorrect.

According to [20, 21] the one-parameter equation may approximate the neck profile for any hardening curve of the material and the instant of the test:

$$\rho = \frac{d_1}{2} - \frac{d_1 - d}{2} \cdot \left(1 + \frac{z^2}{c}\right)^{-1}, \quad (4)$$

where ρ and z are the radial and axial coordinates of points on the specimen surface, d_1 is the diameter of specimen when the neck starts forming and c is the material coefficient. The universal equation (4) makes it possible to develop a mathematical model for predicting the radius of curvature R without performing direct measurements. This work is devoted to this problem.

2. The model of necking the specimen

Let consider the specimen having the original gauge length L_0 , the parallel length L_c and the diameter of the original gauge length d_0 (figure 1). We divide the tension of specimen into two stages. The first stage corresponds to the uniform specimen deformation (figure 2), and the second stage – to the neck formation (figure 3).

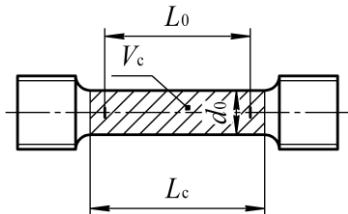


Figure 1. The specimen taken for the model development.

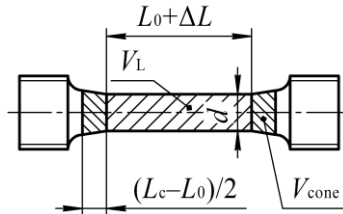


Figure 2. The stage of uniform deformation.

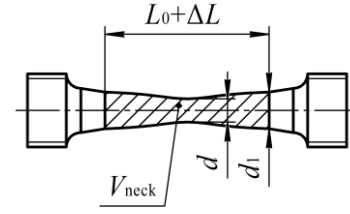


Figure 3. The stage of the neck formation.

We assume that at the first stage the part of deformation is uniformly developed along the gauge length:

$$L = L_0 + \Delta L, \quad (5)$$

where ΔL is the specimen elongation. The other part of deformation is developed into two conical parts having the length

$$L_{\text{cone}} = \frac{L_c - L_0}{2}. \quad (6)$$

We also assume that the conical parts are deformed with the constant length and the cross-sections attached to the specimen grip sections do not change the size. This assumption makes it possible to establish the law of the diameter change d at the first stage of testing the specimen. Thus, we consider the volume constancy equation:

$$V_c = V_L + 2V_{\text{cone}}, \quad (7)$$

where V_c is the volume of specimen corresponding to the original parallel length, V_L is the volume corresponding to the current gauge length L and V_{cone} is the volume of specimen conical part. In accordance with the equation (7), the diameter of the specimen at each instant of the uniform deformation satisfies the formula:

$$d = \frac{d_0 \left[L_0 - L_c + \sqrt{9(L_0 + L_c)^2 + 12(L_0 + 2L_c)\Delta L} \right]}{4L_0 + 2L_c + 6\Delta L}. \quad (8)$$

The second stage of the test occurs when the tensile force assumes the maximum value, i.e. the deformation of the specimen starts to localize in the neck. We consider that the whole deformation is concentrated within the gauge length during the neck formation and the shape of the neck corresponds to the equation (4). Thus, knowing the analytical description of the neck shape we can write the equation of the volume balance in the following form:

$$\Delta V = V_L - V_{\text{neck}}, \quad (9)$$

where ΔV is the material volume displaced along the specimen during tension and V_{neck} is the volume of the specimen into the neck. We represent equation (9) in the following form:

$$\frac{\pi d_1^2}{4}(\Delta L - \Delta L_1) = \frac{\pi d_1^2}{4}(L_0 + \Delta L) - \pi \int_{-(L_0 + \Delta L)/2}^{+(L_0 + \Delta L)/2} \left[\frac{d_1}{2} - \frac{d_1 - d}{2} \cdot \left(1 + \frac{z^2}{c} \right)^{-1} \right]^2 dz, \quad (10)$$

where ΔL_1 is the elongation of the specimen corresponding to the maximum tensile force and d_1 is the diameter of specimen when the neck starts forming. The diameter d_1 is determined by equation (8) for the specimen elongation equal to ΔL_1 .

The numerical solving of the equation (10) allows to find the values of parameter c corresponding to each specific elongation ΔL . We should note that it is necessary to measure the diameter d over the entire stage of the neck formation in order to take into account the influence of the material properties on the neck dimensions. Based on the values of the parameter c , it is possible to predict the radius of curvature of the resulting neck [20, 21]:

$$R = \frac{2c}{d_1 - d}. \quad (11)$$

The predicted radius of the neck curvature R allows calculating the values of the correction coefficient K in accordance with the equation (3) at each instant of the test. Then the material hardening curve can be defined in accordance with equations (1) and (2).

3. The validation of the proposed necking model

Evaluating the accuracy of the proposed mathematical model that allows identifying hardening curve of the material without the need of the direct neck profile measuring was performed by simulating the tensile process in the Deform-2D software. The use of the computer simulation for the research is able to consider any hardening curve of the material and to compare the hardening curve obtained using the proposed mathematical model with it.

We carried out the simulation experiment on five-fold specimens corresponding to the Russian standard GOST 1497. Specimens had the original diameter of the gauge length equal to 6 mm and the parallel length equal to 36 mm. For the simulation, we considered four types of hardening laws: two materials m1 and m2 had a hardening effect only, one material m3 had a constant flow stress and the material m4 had a hardening effect for small strain values and a softening effect for larger strains. The hardening laws of the test materials are the following:

$$\sigma_{m1} = 600\varepsilon^{0.5}; \quad (12)$$

$$\sigma_{m2} = 400\varepsilon^{0.35} + 400; \quad (13)$$

$$\sigma_{m3} = 950; \quad (14)$$

$$\sigma_{m4} = 600\varepsilon^{0.5} \exp(-\varepsilon). \quad (15)$$

Figure 4 presents a comparison of the actual specimen profiles obtained by the results of computer simulation with the predicted profiles obtained on a basis of the proposed mathematical model of necking the specimen. The comparison is available for all considered stress-strain curves. For each test material, we considered four different instants of the test. They corresponded to the change in the diameter d of approximately 0.5 mm. Specimen profiles were defined throughout the parallel length section.

As you can see from figure 4, the accuracy of predicting the whole specimen profile is quite high. You can observe the greatest discrepancy between the actual and predicted profiles in the transition area from the parallel length to the gauge length for the material m1 and the material m4. This is due to the strong hardening effect of these materials when even the grips sections of the specimen begin to deform. In addition you can see from figure 4 that the predicted specimen profiles lie a little wider than the actual profiles for the strongly hardening material m1 and a little narrower for the material m3 having no

hardening effect at all. Thus, the proposed mathematical model of necking the specimen gives an average result for the entire range of the materials hardening curves. For the hardening law of the material m2 that is most common in real tests, the model gives the best result for predicting the specimen profile.

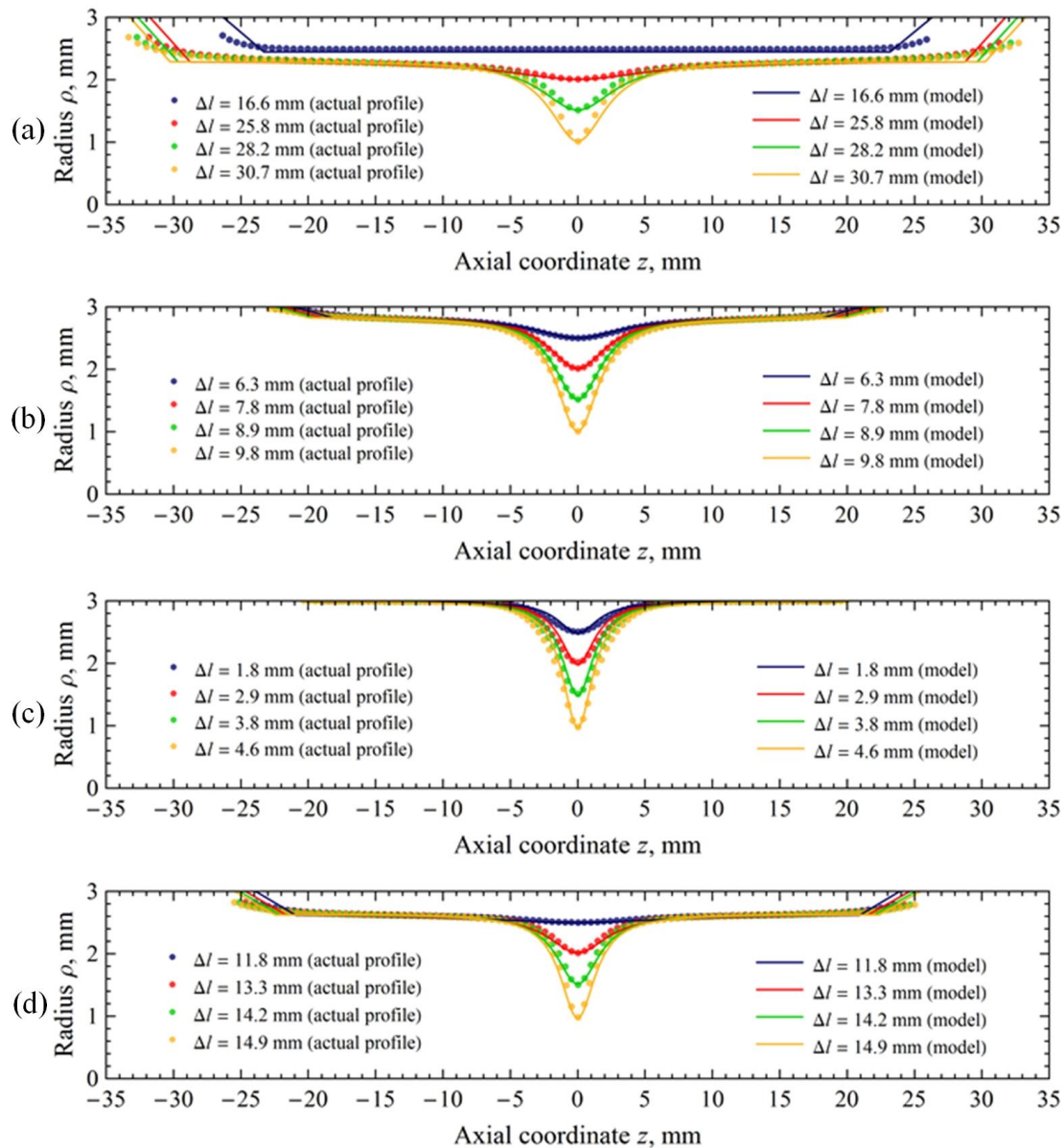


Figure 4. Actual specimen profiles (dots) in comparison with the predicted ones (lines): (a) – the test material m1, (b) – m2, (c) – m3 and (d) – m4.

Based on the described above mathematical model of the neck formation, we defined the hardening curves of the test materials in comparison with the hardening curves that were earlier inputted in the software Deform-2D when the simulation problems were formulated. To calculate the equivalent stresses we used the model of stress distribution in the neck according to Ostsemin. Figure 5 shows the results of calculations of the equivalent stresses as well as the true ones. We determined the true stresses acting in the specimens by equation (1) assuming the correction coefficient K to be constant and equal to 1.

As you can see from figure 5, the equivalent stresses calculated in accordance to the proposed mathematical model are closer to the proper stress values than the true stresses calculated without taking

into account the influence of an inhomogeneous stress distribution in the neck while the methods of direct measuring the neck profile are not available. The maximum deviations of the true and equivalent stresses from the actual ones are respectively: 30.9 and 12.1% for the material m1, 52.8 and 9.6% for the material m2, 64.1 and 14.9% for the material m3 and 89.5 and 29.9% for the material m4. You can observe the greatest deviations for the material m3 and the material m4 at the softening stage. This is most likely because the deformation localizes in the neck more intensively while there is no material hardening. In the case of hardening materials, the model gives more good convergence.

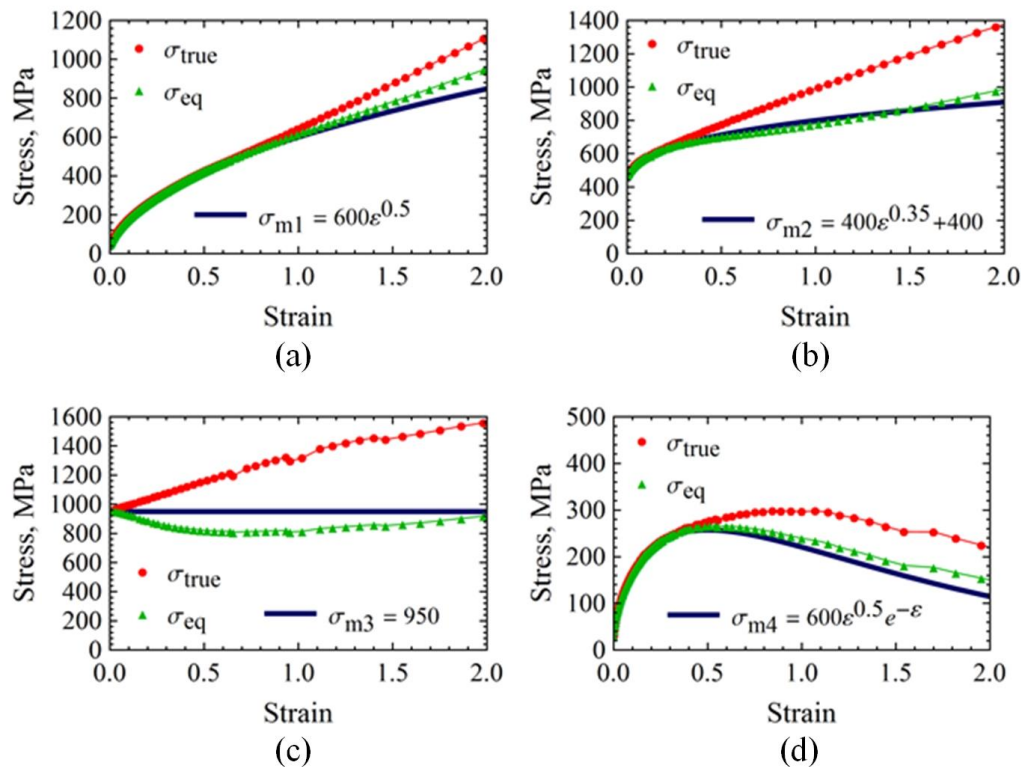


Figure 5. The stress-strain curves in equivalent and true coordinates in comparison with the proper hardening curves: (a) – the test material m1, (b) – m2, (c) – m3 and (d) – m4.

4. Conclusions

The mathematical model of necking the specimen taking into account the effect of grips sections allows predicting the radius of the neck curvature and calculating the equivalent stresses acting in the specimen. The results of computer simulations show that identifying the stress-strain curves on a basis of the proposed model is more preferable than the calculation of the true stresses used in the absence of the possibility to perform the direct measurements of the neck profile.

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